Solution Sample

Electromagnetic (I)
1st Midterm Exam.

College of Electronic Technology
Department of Communications

Date: 02/10/2022
Time: 90 min.

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Answer the following questions:

Q1) Consider the following vectors:

$\vec{A} = \hat{a}_x - \hat{a}_y + 2\hat{a}_z, \quad \vec{B} = \hat{a}_y + \hat{a}_z, \quad \vec{C} = -2\hat{a}_x + 3\hat{a}_z$

Find:

1. $\vec{A}.\vec{B}$
2. $|\vec{A} - \vec{B}|$
3. θ_{AB}
4. *Prove that* : $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$

① $\vec{A}.\vec{B} = (-1+2) = \cancel{+}1 = \underline{\underline{1}}$

② $|\vec{A}-\vec{B}| = |\hat{a}_x - 2\hat{a}_y + \hat{a}_z| = \sqrt{1+4+1} = \underline{\underline{\sqrt{6}}}$

③ $\theta_{AB} = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+1+4}\sqrt{1+1}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{2}}\right) = \cos^{-1}\left(\frac{1}{2\sqrt{3}}\right) = 73.2$

③

$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & 1 \\ -2 & 0 & 3 \end{vmatrix}$$

$\vec{B}\times\vec{C} = 3\hat{a}_x - 2\hat{a}_y + 2a_z$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & -1 & 2 \\ 3 & -2 & 2 \end{vmatrix}$$

$\vec{A}\times(\vec{B}\times\vec{C}) = 2\hat{a}_x + 4\hat{a}_y + \hat{a}_z$

$\vec{A}.\vec{C} = -2+6 = \underline{\underline{4}}$

$\vec{B}(\vec{A}.\vec{C}) = 4(\hat{a}_y + \hat{a}_z) = 4\hat{a}_y + 4\hat{a}_z$

$\vec{A}.\vec{B} = \underline{\underline{1}}$

$\vec{C}(\vec{A}.\vec{B}) = -2\hat{a}_x + 3\hat{a}_z$

$\vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B}) = 2\hat{a}_x + 4\hat{a}_y + \hat{a}_z$

$=$

Q2) Determine the area of a the spherical strip shown in figure Q2 , where the strip is a section of a sphere of radius 3 cm.

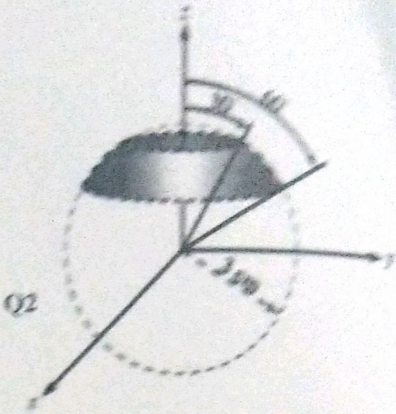

Figure Q2

$\vec{ds} = r^2 \sin(\theta)\, d\theta\, d\phi \;\; \hat{a}_r \qquad , r = 3\,cm$

$S = \iint r^2 \sin(\theta)\, d\theta\, d\phi$

$S = r^2 \int_{30°}^{60°} \sin(\theta)\, d\theta \int_{0}^{2\pi} d\phi$

$S = (3)^2\,(2\pi)\;\; -\cos(\theta)\Big|_{30°}^{60°} = 18\pi \left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right] = \underline{9\pi(\sqrt{3}-1)\,cm^2}$

$= 20.7\,cm^2$

Q3) Transform the vector $\vec{A} = y\hat{a}_x + x\hat{a}_y + \frac{x^2}{\sqrt{x^2+y^2}}\hat{a}_z$,From Cartesian to cylindrical coordinates.

$$\begin{bmatrix} A_\rho \\ A_\rho \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x \\ \frac{x^2}{\sqrt{x^2+y^2}} \end{bmatrix}$$

$A_\rho = y\cos(\phi) + x\sin(\phi)$

$A_\phi = -y\sin(\phi) + x\cos(\phi)$

$A_z = \frac{x^2}{\sqrt{x^2+y^2}}$

$A_\rho = \rho\sin(\phi)\cos(\phi) + \rho\cos(\phi)\sin(\phi) = \underline{2\rho\sin(\phi)\cos(\phi) = \rho\sin(2\phi)}$

$A_\phi = -\rho\sin^2(\phi) + \rho\cos^2(\phi) = \rho(\cos^2(\phi) - \sin^2(\phi)) = \rho\cos(2\phi)$

$A_z = \frac{(\rho\cos(\phi))^2}{\rho} = \frac{\rho^{\cancel{2}}\cos^2(\phi)}{\cancel{\rho}} = \rho\cos^2(\phi)$

$\underline{\vec{A} = \rho\sin(2\phi)\,\hat{a}_\rho + \rho\cos(2\phi)\,\hat{a}_\phi + \rho\cos^2(\phi)\,\hat{a}_z}$

Q4) For the vector field $\vec{G} = \frac{5r^2}{4}\,\hat{a}_r$, Determine the following:

a) The flux of vector $\vec{G}$ through the closed surface shown in figure Q4.

b) Verify Divergence's theorem for the given vector field.

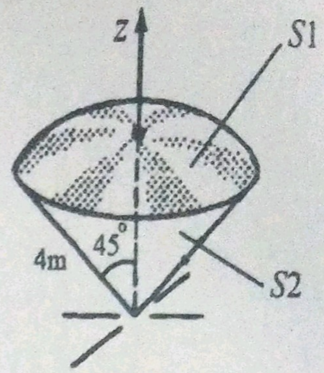

Figure Q4

$$\Psi_1 = \iint \vec{G}\cdot\vec{ds_1} \Rightarrow \vec{ds_1} = r^2\sin(\theta)\,d\theta\,d\phi\ \hat{a}_r \ ,\ \underline{r = 4\text{ m}}$$

$$\Psi_1 = \iint \frac{5r^4}{4}\sin(\theta)\,d\theta\,d\phi$$

$$\Psi_1 = \frac{5(4)^4}{4}\int_0^{45^\circ}\sin(\theta)\,d\theta\int_0^{2\pi}d\phi = 5(4)^3\ -\cos(\theta)\Big|_0^{45^\circ}\ (2\pi)$$

$$\Psi_1 = 640\pi\left[\cos(0) - \cos(45^\circ)\right] = \underline{640\pi\left(1 - \frac{1}{\sqrt{2}}\right) = 588.89}$$

$$\Psi_2 = \iint \vec{G}\cdot ds_2\ ,\ \vec{ds_2} = r\sin(\theta)\,dr\,d\phi\ \hat{a}_\theta$$

$\Psi_2 = 0$, Because $\vec{G}$ is only in direction of $\hat{a}_r$
and $\vec{s}$ is " " " of $\hat{a}_\theta$

$$\Psi_t = \Psi_1 + \Psi_2 = \underline{588.89} = \oiint_S \vec{G}\cdot\vec{ds}$$

$\nabla\cdot\vec{G}$ $\Rightarrow$ G is only in $\hat{a}_r$ direction $\Rightarrow \nabla\cdot\vec{G} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{5r^2}{4}\right)$

$$\nabla\cdot\vec{G} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{5r^4}{4}\right) = \frac{1}{r^2}\left(\frac{20r^3}{4}\right) = \underline{5r}$$

$$\iiint_V \nabla\cdot\vec{G}\,dv = \iiint 5r^3\sin(\theta)\,dr\,d\theta\,d\phi$$

$$= 5\int_0^4 r^3\,dr\int_0^{45^\circ}\sin(\theta)\,d\theta\int_0^{2\pi}d\phi$$

$$= 10\pi\left(\frac{r^4}{4}\right)\Big|_0^4\left(-\cos(\theta)\right)\Big|_0^{45^\circ} = 10\pi(4)^3\left(\cos(0) - \cos(45^\circ)\right)$$

$$\iiint_V \nabla\cdot\vec{G}\,dv = 640\pi\left(1 - \frac{1}{\sqrt{2}}\right) = \underline{588.89}\ ,\quad \oiint_S \vec{G}\cdot\vec{ds} = \iiint_V \nabla\cdot\vec{G}\,dv$$

$$\underline{588.89 = 588.89}\ \checkmark\#$$